ERRATA TO FIRST COHOMOLOGY OF ANOSOV ACTIONS OF HIGHER RANK ABELIAN GROUPS AND APPLICATIONS TO RIGIDITY

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Proposition 4.9 on p.153 of [1] is wrong. This invalidates the given proof of Theorem 2.9 b. The result itself however remains true due to the following additional argument. We use the notations from [1].

Lemma *Let α be a standard Anosov $\mathbb{R}^k$ action on N and f, g Hölder functions on N. Then the matrix coefficients $\langle a\, f, g\rangle$ decay exponentially fast with $a \in \mathbb{R}^k$.*

Proof: If α is a Weyl chamber flow, this follows from the exponential decay of Hölder vectors for unitary representations of semisimple Lie groups of the non-compact type established by D. Kleinbock and G. Margulis in [2, Appendix].

For toral Anosov actions, the claim follows from the fact that Fourier coefficients of Hölder functions with Hölder exponent θ decay decay like $1/n^\theta$ and the fact that the dual action of $\mathbb{Z}^k \subset \mathbb{R}^k$ on multi-indices of Fourier coefficients moves the indices away from 0 exponentially fast (cf. [1, proof of Proposition 4.2].

The remaining cases are subsequent fiber bundle extensions of Weyl chamber flows and/or suspensions of toral actions by toral actions. One can argue as in Section 4.3. Inductively, the contribution to the matrix coefficients from the base decays exponentially fast. On the toral fibers, we can use again Fourier analysis as in 4.3 as $\| \rho(\gamma_{a^k,p})\, I \|$ grows exponentially fast uniformly in p (where p runs over a suitable fundamental domain). $\diamond$

Now we can prove Theorem 2.9 b as follows. As in the smooth case, the exponential decay implies that there always is a distribution solution P to the cohomology equation for the Hölder cocycle β. Let us show that P is a Hölder function. Let $N = M \setminus G/\Gamma$ where M is a compact subgroup of G which commutes with the abelian subgroup A of G which induces the $\mathbb{R}^k$-action α on N. We may lift P to an M-invariant distribution on G/Γ. If n is in the expanding horospherical subgroup N^+ for a then $P - n\, P = \sum_{k=0}^{\infty} a^k f - n\, a^k f$. Since $a^k f - n\, a^k f(x) = f(a^{-k}x) - f(a^{-k}\, n^{-1}x)$ and $d(a^{-k}x, a^{-k}\, n^{-1}x)$ decays exponentially fast, we see that $P - n\, P$ defines a continuous function on G/Γ which depends continuously on $n \in N^+$. Since $P = -\sum_{k=-1}^{-\infty} a^k f$, the same holds for n in the contracting horospherical group N^-. Note that P solves the coboundary equation for all of $\mathbb{R}^k$ and is also M-invariant. Since N^+, N^-, A and M generate G, we see that for all $g \in G$, $g\, P - P$ is a continuous function depending continuously on $g \in G$. Set $h(g) = (g^{-1}\, P - P)(1)$. Then h is a continuous function on G/Γ which is also M invariant. Since for all $b \in A$, $(b\, h - h)(g) = (g^{-1}\, b\, P - P)(1) - (g^{-1}\, P - P)(1) = (g^{-1}(b\, P - P))(1) = \beta(b, g)$, h is a continuous coboundary for β. By [1, Theorem 2.10], h is Hölder.

References

[1] A. Katok and R. J. Spatzier, *First Cohomology of Anosov Actions of Higher Rank Abelian Groups and Applications to Rigidity* , Publ.IHES **79** (1994), 131–156.

[2] D. Kleinbock and G. A. Margulis, *Bounded orbits of nonquasiunipotent flows on homogeneous spaces*, Sinaĭ's Moscow Seminar on Dynamical Systems, 141–172, Amer. Math. Soc. Transl. Ser. 2, 171, Amer. Math. Soc., Providence, RI, 1996.